Understanding spin Hall effects from the motion in SU(2)×U(1) fields

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We derive the classical counterpart of a previously obtained quantum mechanical covariant "continuitylike" equation for the spin density, and present an intuitive picture for elucidating the nonconservation of the spin current. This reveals the equations of motion for a particle with spin under the Yang-Mills field (in certain semiconductors) as well as the Maxwell field, from which the condition for an infinite spin relaxation time is drawn out directly. As a concrete example, we discuss the procession of the spin orientation in spin Hall effect with the so called ReD field, which undergoes a circle with the frequency dependent on both the strength of the spin-orbit coupling and the initial velocity. The anti-commutation of the Pauli matrices is found to be crucial in simplifying the equations of motion in the view of quantum mechanism of the same topics.

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The spin Hall effect [1, 2] is regarded to be practically useful for the rapidly developing field of spintronics [3], in which the spin manipulation, polarization as well as the detection play crucial roles. Theoretically, the spin Hall effect was predicted for p-type semiconductors [4] where the up and down spin particles drift in opposite directions due to an "effective magnetic field" originating from the Berry phase curvature, and a constant spin Hall conductivity was speculated for n-type semiconductors [5] with the help of either the Bloch equations or the Kubo formula. Although the absence of the Hall voltage makes it difficult to detect the pure spin Hall effect directly, the spin accumulation in nonmagnetic semiconductors has been observed experimentally by two groups [6, 7] recently.

The nonconservation [8, 9] of the spin density in the presence of spin-orbit couplings brings about some difficulties relevant to the theoretical analyses [9, 10, 11], such as the definition of the spin current. A covariant form for the continuitylike equation for spin density and current was given [9] in the terminology of Yang-Mills gauge potentials, and it was shown [12] to paly an essential role in guaranteeing the consistency of a generalized Kubo formula for the linear response to non-Abelian fields. Since the work mentioned above are all based on quantum mechanics, one may ask what is the classical counterpart of the covariant form of the continuitylike equation for the spin density, and what is the classical interpretation for the non-conservation of spin current?

For a semiclassical understanding of the spin Hall effects in a two-dimensional electron system with Rashba coupling, an external electric field was shown to exert a transverse force [2, 13, 14] on a moving spin, where the calculation in Ref. [14] is done on the basis of Ehrenfest principle and the discussion in Ref. [13] is carried out with the help of an intuitive picture that the center of a wave packet gets deflection in the transverse direction. The general form of the forces acting on spin and spin current for a variety of systems considered in current literatures was given in Ref. [9]. The time-dependent Rashba spin-orbit coupling was ever proposed to create a force acting on opposite spins in opposite directions [15]. This makes it possible to produce the Yang-Mills field in certain semiconductors according to the viewpoint of our formulism [9]. It is natural to ask whether our formulism has advantage in making sense of the roles that spin and charge play in spin Hall effect. It is therefore an emergent task to elucidate the aforementioned issues via a unified picture.

In this letter, we investigate the motion of a particle with spin in the Yang-Mills field from a classical point of view. We start with a revisit to the classical counterpart of the continuitylike equation for the spin-current density. Such a topic was ever investigated by various authors [16] but has not been correctly exposed yet. We derive the same form as ever proposed quantum-mechanically in the previous paper [9], which presents a clear picture of the non-conservation of the spin current. On the basis of this result, we obtain the equations of motion for a particle with spin in the presence of the Yang-Mills field as well as the Maxwell field. A straightforward condition for an infinite spin relaxation time is drawn out. Then we discuss the spin Hall effect as a concrete example and obtain the trajectory of the spin for the equal strength of Rashba and Dresselhaus couplings. We also discuss the quantum counterpart of the equations of motion and indicate that the anti-commutation relation of the Pauli-matrices makes the problem easy to solve.

We start with considering a moving top (classical analogy of spin) which rotates at a certain rate. A continuum constituted by such kind of tops is completely characterized by a local velocity field $\mathbf{v}(\mathbf{r},t)$, a local particle-density field $\rho(\mathbf{r},t)$ together with a local alignment field $\vec{N}(\mathbf{r},t)$ [17]. As illustrated in Fig. (1), the time evolution of $\vec{N}(\mathbf{r},t)$ is determined by comparing $\vec{N}$ at different times at the same place, i.e., $\vec{N}(\mathbf{r},t+\Delta t)-\vec{N}(\mathbf{r},t)=\vec{\omega}\times\vec{N}(\mathbf{r},t)\Delta t$, while its spatial deviation is determined by comparing $\vec{N}$ at different places simultaneously, i.e., $\vec{N}(\mathbf{r}+\Delta x_i,t)-\vec{N}(\mathbf{r},t)=\vec{\Omega}_i\times\vec{N}(\mathbf{r},t)\Delta x_i$. Hereafter, the

FIG. 1: (color on line) The left scheme depicts the time-evolution of $\vec{N}$ at a point $\mathbf{r}$; the right scheme illustrates the spatial deviation of $\vec{N}$'s by comparing the fields at two neighborhood points $\mathbf{r}$ and $\mathbf{r}+\Delta x_i$ for which the parallel translation is inevitable.

Latin indices run from 1 to 3, and the repeated indices are summed over. The vector fields $\vec{\omega}$ and $\vec{\Omega}_i$ are natural consequences of $\vec{N}$ being a unit vector, then we have

$$\begin{aligned}\frac{\partial}{\partial t}\vec{N}(\mathbf{r},t) &= \vec{\omega}(\mathbf{r},t)\times\vec{N}(\mathbf{r},t),\\ \frac{\partial}{\partial x_i}\vec{N}(\mathbf{r},t) &= \vec{\Omega}_i(\mathbf{r},t)\times\vec{N}(\mathbf{r},t).\end{aligned} \tag{1}$$

By making use of these two relations together with the density conservation $\frac{\partial\rho}{\partial t}+\frac{\partial j_i}{\partial x_i}=0$, we obtain a continuitylike equation

$$\big(\frac{\partial}{\partial t}-\vec{\omega}\times\big)\vec{\sigma}+\big(\frac{\partial}{\partial x_i}-\vec{\Omega}_i\times\big)\vec{J}_i=0, \tag{2}$$

as long as the nature definitions of spin density $\vec{\sigma}=\rho\,\vec{N}$ and spin-current density $\vec{J}_i=\rho\, v_i\,\vec{N}$ are employed. Comparing with the quantum mechanical results [9], one can recognize that $\vec{\omega}$ and $\vec{\Omega}_i$ correspond to the Yang-Mills gauge potentials $\eta\vec{\mathcal{A}}_0$ and $-\eta\vec{\mathcal{A}}_i$, respectively, which have been shown [9] to describe the spin-orbit coupling such as Rashba [18], Dresselhaus [19] coupling and etc. As we are aware, a clear and direct physical meaning of the non-conservation of spin current has not been exposed before.

The above results are suggestive for us to elucidate the equations of motion for a charged vector $\vec{n}$ of constant magnitude (classical analogy of a charged particle with spin) moving in the space undergoing both the Maxwell and Yang-Mill fields

$$\begin{aligned}&\frac{d\vec{n}(t)}{dt}=\eta(\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i)\times\vec{n}(t),\\ &m\frac{dv_i}{dt}=\vec{\mathcal{E}}_i\cdot\vec{n}(t)+eE_i+\epsilon_{ijk}v_j\big(\vec{\mathcal{B}}_k\cdot\vec{n}(t)+eB_k\big).\end{aligned} \tag{3}$$

The first equation for the spin orientation is obtained from Eq.(1) by adopting $\vec{N}(\mathbf{r},t)=\vec{n}(t)\delta(\mathbf{r}-\tilde{\mathbf{r}}(t))$ with $\tilde{\mathbf{r}}(t)$ being the trajectory of a single particle. The second equation is due to the fact that the translational motion is governed by both the Lorentz force caused by the Maxwell fields E_i and B_i and the force by the Yang-Mills fields $\vec{\mathcal{E}}_i$ and $\vec{\mathcal{B}}_i$ which are matrix-valued vectors [9].

Hereafter, $\vec{n}(t)$ is written as $\vec{n}$ for simplicity. To avoid ambiguity, we need to set up a picture with two distinct spaces. One is the SU(2) Lie algebra space (we call it spin space hereafter) in which the Yang-Mills gauge potentials and fields are defined, the other is the conventional spatial space. The "coordinate bases" of the former are $\{\hat{\tau}^1,\hat{\tau}^2,\hat{\tau}^3\}$ with $2\hat{\tau}$ referring to the Pauli matrices, e.g., $\mathbb{B}_i=\mathcal{B}_i^1\hat{\tau}^1+\mathcal{B}_i^2\hat{\tau}^2+\mathcal{B}_i^3\hat{\tau}^3$, while those of the later are $\{\mathbf{e}_x,\mathbf{e}_y,\mathbf{e}_z\}$, e.g., $\mathbf{v}=v_x\mathbf{e}_x+v_y\mathbf{e}_y+v_z\mathbf{e}_z$. In the language of the gauge potentials, the Yang-Mills "electric" and "magnetic" fields can be expressed as

$$\begin{aligned}\vec{\mathcal{E}}_i &= -\partial_0\vec{\mathcal{A}}_i-\partial_i\vec{\mathcal{A}}_0+\eta\vec{\mathcal{A}}_0\times\vec{\mathcal{A}}_i,\\ \vec{\mathcal{B}}_i &= \epsilon_{ijk}\partial_j\vec{\mathcal{A}}_k+\frac{\eta}{2}\epsilon_{ijk}\vec{\mathcal{A}}_j\times\vec{\mathcal{A}}_k.\end{aligned} \tag{4}$$

It is worthwhile to note that the non-Abelian fields can be non-vanishing even when the gauge potentials are constant.

In the view of the analytical mechanics, the equations of motion (3) can be formulated in the Hamiltonian formulism with $H=\frac{1}{2m}(p_i-eA_i-\eta\vec{\mathcal{A}}_i\cdot\vec{n})^2+eA_0+\eta\vec{\mathcal{A}}_0\cdot\vec{n}$. As a standard procedure, one needs to choose two sets of canonical coordinates and their conjugations (canonical momentums), namely, (ϕ,n_z) with $\phi=\tan^{-1}(-n_x/n_y)$ and (r_i,v_i). One pair of the canonical equations can be easily derived

$$\begin{aligned}\frac{d\phi}{dt} &= \frac{\partial H}{\partial n_z}=\frac{\eta}{n_\perp^2}\big((\mathcal{A}_0^z-v_i\mathcal{A}_i^z)n^2-n_z(\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i)\cdot\vec{n}\big),\\ \frac{dn_z}{dt} &= -\frac{\partial H}{\partial\phi}=\big(\eta(\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i)\times\vec{n}\big)_z,\end{aligned}$$

with $n_\perp^2=n^2-n_z^2$, which is exactly the first equation of Eq. (3) after some algebra. The other pair of the canonical equation respect to (r_i,v_i) can be shown to be the second one of Eq. (3).

Some immediate consequences can be obtained from Eq. (3). (**i**) The first equation for the time rate of $\vec{n}$ clearly manifests that the vector $\vec{n}$ does not process when it is parallel to $\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i$, or more specially, $\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i=0$. In such cases, the spin orientation keeps unchanged, which results in an infinite spin relaxation time. A typical example is that the infinite spin relaxation time occurs in the $\pm[1,\pm1,0]$ direction when $\vec{\mathcal{A}}_0=0$ and $\alpha=\pm\beta$ (α and β refer to Rashba and Dresselhaus coupling strength, respectively, in the quantum case), one of which was discussed in Ref. [20]. This is analogous to the case in the classical electrodynamics where an electron moving in the uniform orthogonal electromagnetic fields with velocity $\mathbf{v}=\mathbf{E}\times\mathbf{B}/B^2$ does not feel the Lorentz force. (**ii**) In virtue of the coupling between $\vec{n}$ and the Yang-Mills field in the second equation of (3), the time rate of $\vec{n}$ leads to the time-dependent effective fields even when the Yang-Mills fields are time-independent, which is quite different from the motion in the Maxwell field.

For many real physical systems, the gauge potentials $\vec{\mathcal{A}}_i$ are constant. If we focus on the contribution of the

Yang-Mills field and set the Maxwell one to be zero, the second equation of (3) reduces to

$$m\frac{dv_i}{dt} = -\eta\vec{\mathcal{A}}_i\cdot(\vec{\mathcal{A}}_0 - v_j\vec{\mathcal{A}}_j)\times\vec{n}, \tag{5}$$

which gives rise to a relation between $\vec{n}$ and v_i

$$v_i(t) = -\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}(t) + C_i, \tag{6}$$

where $C_i = v_{0i} + \frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}_0$ are determined by the initial values $v_i(0) = v_{0i}$ and $\vec{n}(0) = (n_{0x}, n_{0y}, n_{0z})$. Consequently, one only needs to solve one equation,

$$\frac{d\vec{n}}{dt} = \eta(\vec{\mathcal{A}}_0 - C_i\vec{\mathcal{A}}_i)\times\vec{n} + \frac{\eta}{m}(\vec{\mathcal{A}}_i\times\vec{n})(\vec{\mathcal{A}}_i\cdot\vec{n}). \tag{7}$$

As a concrete example, we consider a two-dimensional system with uniform Yang-Mill gauge potentials [9]

$$\vec{\mathcal{A}}_x = \frac{2m}{\eta^2}(\beta,\,\alpha,\,0),\quad \vec{\mathcal{A}}_y = -\frac{2m}{\eta^2}(\alpha,\,\beta,\,0),\quad \vec{\mathcal{A}}_0 = 0, \tag{8}$$

which correspond to Rashba and Dresselhaus couplings in certain semiconductors. Accordingly, the equations of motion are explicitly written as

$$\begin{aligned}
\frac{dn_x}{dt} &= -\frac{2m}{\eta}n_z\Big[\alpha C_1 - \beta C_2 - \frac{4\alpha\beta n_x + 2(\alpha^2+\beta^2)n_y}{\eta^2}\Big],\\
\frac{dn_y}{dt} &= \frac{2m}{\eta}n_z\Big[\beta C_1 - \alpha C_2 - \frac{4\alpha\beta n_y + 2(\alpha^2+\beta^2)n_x}{\eta^2}\Big],\\
\frac{dn_z}{dt} &= \frac{2m}{\eta}n_x\Big[\alpha C_1 - \beta C_2 - \frac{4\alpha\beta n_x}{\eta^2}\Big]\\
&\quad -\frac{2m}{\eta}n_y\Big[C_1\beta - C_2\alpha - \frac{4\alpha\beta n_y}{\eta^2}\Big].
\end{aligned} \tag{9}$$

For $\alpha = \beta$ which is called ReD field in Ref. [20], we can solve these equations analytically:

$$\begin{aligned}
n_x &= -a\sin(\omega t+\varphi) + \frac{1}{2}(n_{0x}+n_{0y}),\\
n_y &= a\sin(\omega t+\varphi) + \frac{1}{2}(n_{0x}+n_{0y}),\\
n_z &= \sqrt{2}a\cos(\omega t+\varphi),
\end{aligned} \tag{10}$$

where $a = \sqrt{n_{0z}^2/2 + (n_{0x}-n_{0y})^2/4}$ and $\tan\varphi = (n_{0y} - n_{0x})/(\sqrt{2}\,n_{0z})$ are determined by the initial conditions. It is clear that the tip of $\vec{n}$ experiences a cyclotron rotation with frequency $\omega = \frac{2\sqrt{2}m\alpha}{\eta}(v_{0x} - v_{0y})$, which is shown in Fig. (2) for the initial condition $\vec{n}_0 = (1,0,0)$. The instantaneous velocity solved from Eq. (6) is just its initial value $v_x = v_{0x}$ and $v_y = v_{0y}$, i.e., the electron undergoes a motion with uniform velocity. This is due to the time-dependent parts of n_x and n_y only differ from each other by a minus sign. Specially, when $v_x = v_y$, the spin vector $\vec{n}$ does not process since $\omega = 0$, which recovers the result in Ref [20].

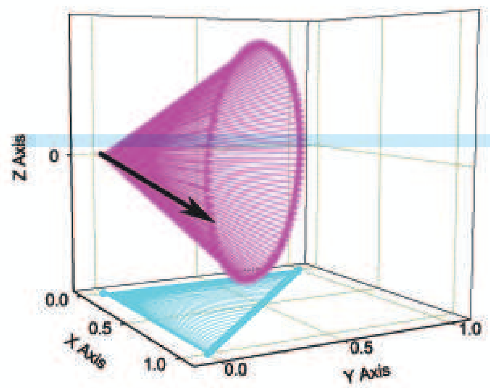

FIG. 2: (color on line) The rotation of $\vec{n}$ is depicted with the initial conditions $\vec{n}_0 = (1,0,0)$, where the x-y projection is also shown to elucidate this motion.

When an external electric field $\vec{E} = (E_x, E_y)$ is applied, which mimics to the usual spin Hall effect in current literature, we obtain $v_i(t) = -\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}(t) + C_i + \frac{e}{m}E_i t$. The equation of motion for $\vec{n}$ is almost the same as Eq. (7) if substituting C_i by $\tilde{C}_i(t) = C_i + \frac{e}{m}E_i t$. For the electric field is sufficiently weak, the perturbation theory is applicable and we can expand the spin unit vector $\vec{n}$ in the power of the electric field,

$$\vec{n} = \vec{n}^{(0)} + \vec{n}^{(1)} + \cdots. \tag{11}$$

The 0th-order results take the form of Eq. (10), while the linear corrections are

$$\begin{aligned}
n_x^{(1)} &= -\lambda(E)\ t^2\cos(\omega t+\varphi),\\
n_y^{(1)} &= \lambda(E)\ t^2\cos(\omega t+\varphi),\\
n_z^{(1)} &= -\sqrt{2}\lambda(E)\ t^2\sin(\omega t+\varphi),
\end{aligned} \tag{12}$$

where $\lambda(E) = \alpha e a(E_x - E_y)\sqrt{2}/\eta$. The form of the entire motion for $\vec{n}$ is much similar to the case without external electric fields, except for a time-dependent amplitude $\tilde{a}(t) = [a^2 + \lambda^2 t^4]^{1/2}$ and a phase $\tilde{\varphi}(t) = \varphi + \tan^{-1}(\lambda t^2/a)$. Even though the rotation of $\vec{n}$ is a bit of intricacy, its translational motion only involves one more term linearly dependent on time,

$$\begin{aligned}
v_x &= v_{0x} + \frac{e}{m}E_x t,\\
v_y &= v_{0y} + \frac{e}{m}E_y t.
\end{aligned} \tag{13}$$

Furthermore, we turn to the quantum picture of the motion of a single particle with spin in the background of the Yang-Mills field with the Hamiltonian $\hat{H} = \frac{1}{2m}\big(\hat{p}_i - eA_i - \eta\vec{\mathcal{A}}_i\cdot\vec{\tau}\big)^2 + eA_0 + \eta\vec{\mathcal{A}}_0\cdot\vec{\tau}$ [9]. The time rate of its dynamical momentum $\hat{\pi}_i \equiv \hat{p}_i - eA_i - \eta\vec{\mathcal{A}}_i\cdot\vec{\tau}$ is given by the Heisenberg equation of motion $d\hat{\pi}_i/dt = \frac{1}{i\hbar}[\hat{\pi}_i, H] = eE_i + \eta\vec{\mathcal{E}}_i\cdot\vec{\tau} + \frac{1}{2}\epsilon_{ijk}\{v_j, eB_k + \eta\vec{\mathcal{B}}_k\cdot\vec{\tau}\}$, where

the curve parentheses denote the anti-commutator. Similarly, the equation of motion for the spin operator reads $d\vec{\tau}/dt = \frac{1}{i\hbar}[\vec{\tau}, H] = \eta\vec{\mathcal{A}}_0 \times \vec{\tau} - \frac{\eta}{2}\{v_j, \vec{\mathcal{A}}_j \times \vec{\tau}\}$. Obviously, these equations are the quantum counterpart of Eq. (3) with a $\eta\vec{\tau}$ to $\vec{n}$ correspondence. In the previous classical case, it is difficult to solve the equations of motion since they are coupled to each other. However, things become easier in the quantum case. Substituting $v_j = \hat{\pi}_j/m = \frac{1}{m}(p_j - eA_j - \eta\vec{\mathcal{A}}_j \cdot \vec{\tau})$ into the equation of motion for $\vec{\tau}$, we obtain $d\vec{\tau}/dt = \eta\vec{\mathcal{A}}_0 \times \vec{\tau} - \frac{\eta}{2}\{p_j - eA_j, \vec{\mathcal{A}}_j \times \vec{\tau}\}$ in which the anti-commutation relation $\{\tau^a, \tau^b\} = \frac{1}{2}\delta^{ab}$ has been used. The time evolution of p_j obeys $dp_j/dt = [p_j, H]/i\hbar$. The equations of motion which are coupled to each other in the classical mechanics become decoupled in the quantum case where the anti-commutation relations of $\vec{\tau}$ play a key role. For example, in the conventional spin Hall system, the external electric field is applied to drive the spin current and $dp_i/dt = eE_i$. The equation of motion for $\vec{\tau}$ reduces to the one ever considered in Ref. [5].

In the above, we investigated the motion of a particle with spin in the Yang-Mills field from a classical point of view which differs from that of Ref [16], and presented a much more clear picture for the non-conservation of spin current, which has not been correctly exposed before. We revisited to the classical counterpart of continuitylike equations for spin-current density, which has the same form as we ever proposed quantum-mechanically in a previous paper [9]. With those results, we elucidated the equations of motion for a particle with spin in the presence of Yang-Mills fields as well as the Maxwell fields. Although various authors [5] have considered the equation of motion, the complete form (3) has not been recognized yet. It is worthwhile to note that the appearance of the coupling between the spin orientation and the Yang-Mills fields brings about the time-dependent effective fields even when the Yang-Mills fields are time-independent. In other words, the motion of the particle is greatly affected by the procession of its spin. From one of equations of motion, we can easily obtain the condition when the spin relaxation time is infinite and a special case of our conclusion recovers the previous result discussed by other authors. We have taken the spin Hall system as a concrete case and found that the tip of the spin vector undergoes a cyclotron motion with the frequency determined by the strength of the spin-orbit coupling and the initial velocity of the particle. Furthermore, we reviewed the quantum counterpart of the equations of motion and indicated that the anti-commutation relation of the Pauli matrices makes the equations of motion decoupled, which is thus easy to solve.

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